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Particle-wave duality of solitons: scaling symmetry breaking in soliton scattering, hidden role of the self-interaction energy and the de Broglie video-soliton

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ABSTRACT

Today, the soliton paradigm provides a luxuriant source of inspiration in many areas of fundamental physics and technology: from nonlinear optics and physics of Bose-Einstein condensates (BECs) to monster (rogue) waves in oceans, elementary particle physics and cosmology. Solitons - self-localized robust and long-lived nonlinear solitary waves with properties of real particles - arise in any physical system possessing both nonlinearity and dispersion, diffraction or diffusion (in time or/and space). Today, optical soliton presents the remarkable example in which an abstract mathematical concept has produced a large impact on the real world of high technologies. Owing to its remarkable features, the soliton might appear as an idealized mathematical structure for the description of extended "elementary" particles. The famous principle of particle-wave duality in quantum physics holds that matter and light exhibit the behaviors of both waves and particles, depending on the experiment being performed. Is there a similar analogy for the soliton? What other analogies have not been established so far? The key conceptual result of our work consists in the demonstration of a deep analogy between the Schrödinger soliton tunneling through the classically forbidden potential barrier and the Gamow scenario of quantum mechanical tunneling effect and alpha-particle decay. Guided by this constructive (but obviously only formal) analogy, we reveal a hidden role of the soliton self-interaction ("binding") energy and its dramatic impact on the particle-wave duality of scattered solitons. The solitonic analog of the de Broglie wavelength, "shooting out" of high-energy de Broglie video-solitons from arbitrary N-soliton superposition and phenomena similar to the Ramsauer-Taunsend effect and the Geiger--Nuttall law ought to be expected for solitons.

"The exact space-time images for the dualism of wave-particle must be sought" (Louis de Broglie, Letter to Einstein, 8 March 1954)

1. Introduction

Textbook treatment of the particle-wave duality recognizes that "the existence of corpuscles accompanied by waves has to be assumed in all cases" [1]. This May 2011, marks the 100th anniversary of the most important experiments conducted in the history of science - the Rutherford alpha-particle scattering and the discovery of the atomic nucleus, but so far, the nature of the de Broglie matter wave associated with point-like elementary particle remains one of the most intriguing in physics. It will suffice to mention that, two great founders of quantum mechanics, Louis de Broglie and Erwin Schrödinger, before the end of their days were not satisfied by the adequacy of the physical world that they themselves created: Erwin Schrödinger "disliked the generally accepted dual description in terms of waves and particles, with a statistical interpretation for the waves, and tried to set up a theory in terms of waves only" [2] and Louis de Broglie returned to a direct and real physical interpretation of matter-waves and worked in the foundations of his "theory of the double solutions" and "non-linear wave mechanics" [1]. In this sense they were in solidarity with Einstein who, like them, believed that in an ultimate theory, the particles will be represented by, in the modern terms, soliton-like solutions for the highly nonlinear field equations [3, 4].

"Coming events cast their shadows before them". In the history of science, we may find many examples when outstanding scientists were far ahead of their time. The discovery of solitons dates back to the beautiful Report of Scott Russel [5, 6]. Today, optical soliton presents the remarkable example in which an abstract mathematical concept has produced a large impact on the real world of high technologies: considerable advances have been made in dense WDM with dispersion-managed solitons, soliton supercontinuum generation, and new soliton lasers design [7-15].

Behaved like particle a soliton may be transmitted through a potential difference and reflected from the potential barriers, and, consequently, general questions naturally arise: To what extent the soliton (as the analog of extended "elementary" particle) can be regarded as a classical point-like particle which is governed by Newton's equations of motion? As to whether an internal structure and intrinsic ("hidden") soliton degrees of freedom can show up in the soliton tunneling through the potential barriers and wells? More generally, how the particle-wave duality demonstrates itself for the soliton?

The interaction of solitons with local inhomogeneities is a fundamental problem in physics that appears in the modern literature in different contexts (see, for example, the review of nonlinear tunneling principles and research as it currently stands in [16-24], and references therein). The soliton tunneling effect is far from being trivial, and here, we specially reveal many deep analogies with fundamental effects of nuclear and particle physics.

We show that one further peculiarity of the particle-wave duality (that is yet hidden from us) can be exhibited for solitons: on the one hand, as the self-localized wave object, the soliton, by virtue of the Galilean symmetry, is characterized by its own solitonic analog of the de Broglie wavelength, and on the other hand, as the extended particle-like object, the soliton, because of the nonlinearity, becomes a bound state in its own self-induced trapping potential and, as a consequence, acquires a negative self-interaction ("binding") energy. This form of energy provides the shape and structural stability of solitons as extended objects and, similar to the nuclear binding energy, can be considered as the degree of how strongly the quasi-particles that make up the soliton are bounded together: photons in optical spatial or temporal solitons, or Bose atoms trapped in matter-wave soliton in BEC.

We disclose an amazing analogy between the Schrödinger soliton tunneling effect and quantum mechanical Gamow's model for alpha-particle decay, and inspirited by many remarkable (but, obviously, only formal) analogies with modern Rutherford experiments and nuclear sub-barrier reactions (where the most important innovations are connected with quantum tunneling of a composite particle, in which the particle itself has an internal structure [25]), we point the way to discover novel phenomena for solitons similar to the well-known in atomic and nuclear physics the Ramsauer-Taunsend effect and the Geiger--Nuttall law.

2. The analogy with Gamow's scenario of alpha-particle decay: "solitonic toy model of a nucleus"

The question of the existence of a connecting link between particle-like and wave-like behaviors of solitons has been a puzzling problem for physicists for last decades and remains the most intriguing and important open problem to this day. The most common difficulty (historically started from the earliest works by Hasse and de Moura [26, 27]) is the belief that the soliton in framework of the nonlinear Schrödinger equation (NLSE) model with external potentials

$$i\frac{\partial\psi(\xi,\eta)}{\partial\eta} = -\frac{1}{2}\frac{\partial^2\psi(\xi,\eta)}{\partial\xi^2} - \sigma|\psi(\xi,\eta)|^2\psi(\xi,\eta) + U_{ext}(\xi) \qquad (1)$$

behaves as a classical particle which is governed by Newton's equations of motion. Eq.(1) is written here in standard dimensionless form to emphasize all common analogies between different types of Schrödinger solitons in different fields of science irrespective to physical interpretation of the complex function $\psi(\xi,\eta)$ and physical mechanisms of origin of the nonlinearity [3-28]. In the linear limit $\sigma = 0$, Eq.(1) represents the classic Schrödinger equation (SE) with its probabilistic quantum mechanical interpretation of the wave function $\psi(\xi,\eta)$ under the condition $\int_{-\infty}^{\infty} |\psi(\xi,\eta)|^2 d\xi = 1$. In photonics, the NLSE Eq.(1) is known also as the nonlinear parabolic equation for the slowly varying envelope of the electric field $\psi(\xi,\eta)$ [6-11]. The parameter $\sigma = \pm 1$ separates attractive and repulsive self-interactions (namely, bright and dark, both temporal and spatial solitons).

Without external potential $U(\xi)=0$, the canonical NLSE model [28]

$$i\frac{\partial\psi(\xi,\eta)}{\partial\eta} = -\frac{1}{2}\frac{\partial^2\psi(\xi,\eta)}{\partial\xi^2} - \sigma|\psi(\xi,\eta)|^2\psi(\xi,\eta) \qquad (2)$$

is invariant under the scaling and the Galilean transformations

$$\psi'(\xi,\eta;□) = □\psi(□\xi,□^2\eta) \qquad (3)$$

$$\psi'(\xi,\eta;V) = \psi(\tilde{\xi}V\eta)exp(iV\tilde{\xi}iV^2\eta/2), \qquad (4)$$

which can be applied to all its arbitrary solutions [5-7]. In particular, under the scaling and Galilean symmetry the fundamental bright ($\sigma = +1$) NLSE soliton takes the most general form

$$\psi_s = □sech[□(\tilde{\xi}V\eta)\ ]exp[iV\tilde{\xi}i(\tilde{V}^2□^2)\eta/2)] \qquad (5)$$

with the scaling parameter □ (known also as the soliton form-factor) defining both the soliton amplitude, its size $\xi_s=1/□$ and its spectral distribution; and V is the velocity along the ξ axis. In general, scaling symmetry means that if some object is expanded or reduced in size, the new object has the same properties as the original.

It is straightforward to verify that an arbitrary external potential $U(\xi)$ breaks the scaling symmetry of the NLSE model Eq.(1). That is why, we must expect to find considerable changes (and, probably, surprises) in dynamics of solitons with different form-factors □. It should be also mentioned that soliton form-factors □ define the most important conserved integrals of motion

$$N_q = \int_{-\infty}^{\infty} |\psi(\xi,\eta)|^2 \, d\xi = 2\square \tag{6}$$

$<E> = <T_{kin}> + <E_{int}> + <U_{pot}>$

$$= \langle E \rangle = \frac{\langle \psi | \widehat{H} | \psi \rangle}{\langle \psi | \psi \rangle} = \frac{\int_{-\infty}^{\infty} \left[|\psi_{\xi}(\xi,\eta)|^2 - |\psi(\xi,\eta)|^4 + 2U(\xi)|\psi|^2 \right] d\xi}{2\int_{-\infty}^{\infty} |\psi(\xi,\eta)|^2 \, d\xi}, \tag{7}$$

where the mean kinetic energy

$$<T_{kin}> = \square^2/6 + V^2/2 \tag{8}$$

does not vanish for the soliton (5) being at rest $<T_{kin}>_{V=0} = \square^2/6$ in strong analogy with a quantum mechanical particle being at rest [29]. The mean self-interaction energy $<E_{int}>$ of the bright soliton (5) is given by

$$<E_{int}> = -\square^2/3 \tag{9}$$

so that its absolute value is two times greater then the rest mean kinetic energy: $<E_{int}> = 2 <T_{kin}>_{V=0}$.

In Fig. 1, we show the dependence of the effective potential $U_{eff}=U(\xi)-|\psi(\xi,\eta)|^2$ on the self-interaction energy for solitons, placed, for simplicity, on the top of the barrier. The origin of a deep and narrow hole on the barrier can be understood by looking at the structure of the canonical NLSE (1): the third term in Eq. (1) represents the equivalent self-trapping potential $U_{tr} = -|\psi(\xi,\eta)|^2$ which becomes deeper and deeper when the maximal amplitude of the soliton (and hence, the soliton self-interaction energy $|E_{int}|$ = □²/3) grows, so that the attractive potential well inside the repulsive barrier is lowered into the negative energy region as shown in Fig. 1(c-d).

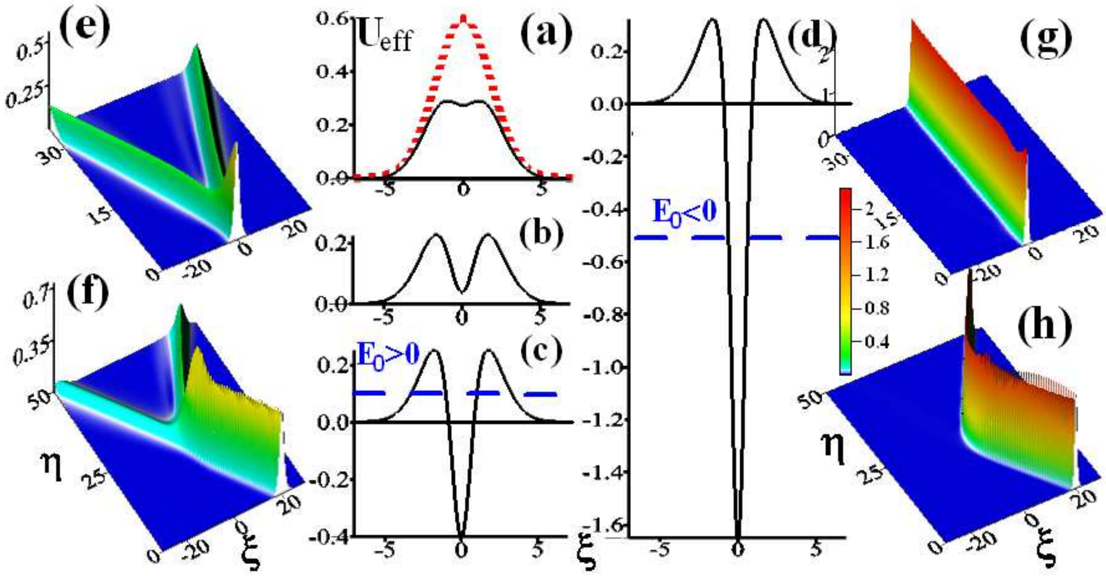


Fig.1. "Hidden" features of nonlinear soliton tunneling effect. (a-d) "Solitonic toy model of a nucleus" given by effective potential for solitons placed on the top of the barrier $U(\xi)=U_0 exp(-\xi^2/\delta^2)$ after choosing the parameters: U_0=0.6, δ=2.5, and (a) □=0.5, (b) □=0.75, (c) □=1.0, (d) □=1.5. (e-f) Soliton decay associated with the Gamow tunneling mechanism from the metastable energy level $E_0>0$ (c) after choosing (e) $V=0$ and (f) $V \cong 1.0$. (g) Soliton "self-trapping" on the top of the repulsive (anti-waveguiding) potential in the parameters region shown in Fig. 1(e) and corresponding to the bound-state (negative) energy level E_0 and its complete Newtonian like reflection (h) from classically forbidden barrier at $V \cong 1.0$.

It should be stressed that the total effective potential (see Fig. 1(c-d)) looks qualitatively like a one-dimensional "toy model of a nucleus" (find analogy in [29], exercise 7.11). Actually, when the effective potential given in our Fig. 1 is compared with the famous Gamow's Fig. 4 for the potential energy of the alpha particle as a function of its separation from the center of the nucleus (for the first time shown in his paper [30]), it is apparent that the soliton behavior in a finite scattering potential $U(\xi)$ demonstrates remarkable analogy with Gamow's model for alpha-particle tunneling. Notice that the exact solution of the soliton tunneling problem can be obtained only numerically, but some quantitative estimations are available from the well known (from the Inverse Scattering Transform method [5-7]) correspondence of the given nonlinear problem to the linear one. Let us consider the linear SE with trapping effective potential $U_{tr} \cong |\psi(\xi,\eta)|^2$ which is produced by the soliton itself

$$i\frac{\partial \psi(\xi,\eta)}{\partial \eta} = -\frac{1}{2}\frac{\partial^2 \psi(\xi,\eta)}{\partial \xi^2} + [U(\xi) - \square^2 \, sech^2(\square\xi)]\,\psi \qquad (10)$$

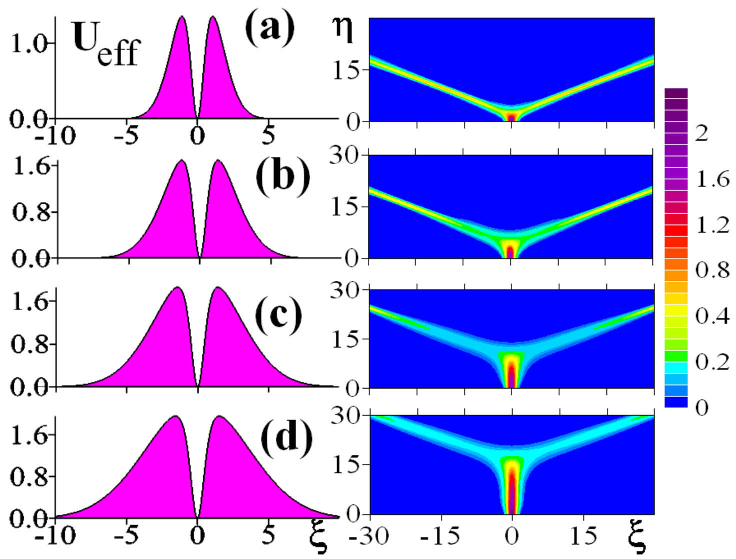
and suppose that the potential barrier can be represented by the slowly varying function of coordinate (in comparison with the soliton width), so that its height is given by $U(\xi) \to U_{max}$ in the region $\xi \to 0$. Eq. (10) with well-defined the energy levels has been known under several different names from the first years of quantum mechanics: SE with symmetric Rosen-Morse or modified Pőschl-Teller potentials [31]. It turns out that under condition $U_{max} = 0$ there exists only one localized solution with only one bound-state energy level $E_s = -\square^2/2$ that is exactly coincident (!) with the fundamental soliton solution Eq. (5) of the canonical NLSE Eq. (2): $\psi_s = \square sech(\square\xi) exp(-iE_s\eta)$.

Under the action of sufficiently strong repulsive external potential (when the soliton self-interaction energy still remains to be small enough $|E_{int}| < 2\, U_{max}/3$), the bound-state energy level E_s (see Fig. 1(d)) can be uplifted into the positive energy region $E_0 = U_{max} - \square^2/2 > 0$, where it is transformed into the metastable level shown in Fig. 1(c) with characteristic life-time dependent on the barrier width. That is why we clarify the physical relation of our problem with the Gamow tunneling mechanism (see Fig. 1(c, e-f)). In opposite case of greater "binding" energies: $|E_{int}| > 2\, U_{max}/3$, the energy level E_0 remains be bounded $E_0 < 0$, and the soliton cannot escape. In other words, the soliton at the bound-state energy level resembles a classical particle in the sense that the soliton maintains its integrity and follows a well defined classical trajectory shown in Fig. 1(d, g-h).

Direct computer experiments confirm all our conclusions. We have computed different scattering scenarios with different soliton parameters and for different profiles of the barriers. In what follows, we define that the main features of the soliton tunneling effect remain the same. There exists the critical strength of the soliton self-interaction energy above which all solitons with kinetic energies $V^2/2 < U_{max}$ are reflected elastically and never transmitted (see Fig. 1(h)). It should be emphasized that Fig. 1(g) presents the case when stable soliton is formed on the top of the repulsive barrier, which in optical applications corresponds to the possibility of self-waveguiding soliton propagation in the anti-waveguiding potential. Notice that in the case of optical spatial solitons, the initial soliton kinetic energy corresponds to the beam incidence angle shown in Fig. 1(h).

It should be emphasized that the analogy with a "toy solitonic model of a nucleus" given in Fig. 1(c-d) not only helps us to better understand the physical mechanism of the Schrödinger soliton tunneling and decay, but also demonstrates much more deeper features. Gamow pioneered his tunneling idea [30] to explain and deduce theoretically the experimental Geiger-Nuttall law, which relates the decay constant of a radioactive isotope with the energy of the alpha particles. Roughly speaking, the greater is the kinetic energy of emitted alpha particles, the smaller is their period of stay (half-lives) inside the nucleus. Similar behavior must be expected for solitons. We have seen that the soliton, like the alpha particle, can be regarded as trapped by a potential barrier that is shown in Fig. 1. In order to escape into the environment, the soliton, just as the alpha particle, must tunnel through the barrier created by its own self-interaction potential as illustrated in Fig. 1. In particular, similar to the Gamow theory, the kinetic energy of emitted solitons can be shown to be independent on the detailed form of the barrier and its thickness.

Somewhat surprisingly, many considerable educational problems associated with the understanding of this statement persist to the present at student learning of quantum mechanics (see [32]). Really, in the Gamow scenario, the main feature of alpha-particle decay consists in the conservation of the total energy of the alpha particles [30]. Gamow specially stressed in his pioneering paper [30] that: "In wave mechanics a particle always has a finite probability, different from zero, of going from one region to another region of the same energy, even through the two regions are separated by an arbitrarily large but finite potential barrier." As follows from our computer simulations presented in Fig. 2(a-d), the kinetic energy of emitted solitons is independent on the barrier thickness if the barrier height remains the same. The "life-time of decaying state" of the soliton is found to be in exponential dependence on the barrier width as we show in Fig. 2(f). If, instead of varying the barrier width, we vary the barrier height, which is analogous to a charge of the nucleus in our "toy model", we obtain the remarkable linear relation between the velocities of tunneling solitons and the barrier height shown in Fig. 2(e). This relation is analogous to the well known to Gamow linear dependence of velocities of emitted alpha particles on the charges of nuclei [30], and Figs. 2 (e,f) reveal important "hidden" analogies with the Geiger-Nuttall law in nuclear physics.



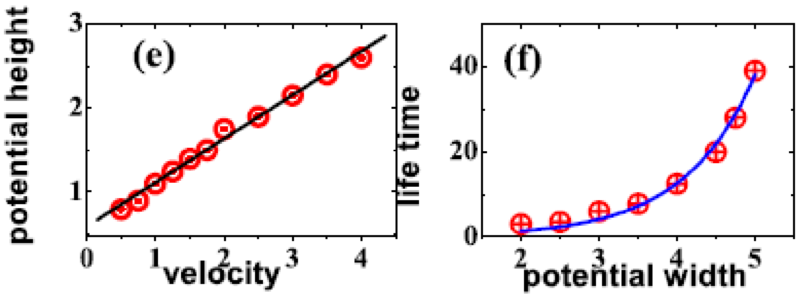


Fig.2 The analogy with Gamow's scenario of alpha-particle decay. (a-d) Dynamics of the soliton decay on the barriers with varying thickness. (e) Linear relation between the velocities of tunneling solitons and the barrier height. (f) Exponential dependence of the "life-time of decaying state" on the barrier width.

3. "Solitonic de Broglie wavelength" and the analog of the Ramsauer-Townsend effect

One can look at the hidden role of the soliton self-interaction energy somewhat differently. It is easy to see that in the framework of the models (1-2) (both linear $\sigma=0$ and nonlinear $\sigma=\pm 1$), the de Broglie wave arises as a consequence of the Galilean symmetry Eq. (3). This fact can be easy understood by exploiting the mathematical equivalence between the linear SE and propagation of light pulses in optical fibers, and extending this analogy onto nonlinear regime. The main information about the "future" of the NLSE soliton is encoded in its phase: the Galilean symmetry provides the appearance of the classic kinetic energy $T_{kin} = V^2/2$ through the term $iV^2\eta/2$ (see Eq. (5)) and the existence of sinus-like modulation with the wavelength analogous to the de Broglie wavelength $\lambda_{dB}=2\pi/V$ through the term $iV\xi$ in Eq. (5). The soliton is an extended object with its own size ξ_s =1/□ and, because of this, its envelope function cuts up the sinus oscillations with wavelength λ_{dB} giving rise to both the solitonic wave packets if $\xi_s > \lambda_{dB}$ (see Fig. 3(b)) and the de Broglie video-solitons without a high-frequency filling if $\xi_s < \lambda_{dB}$ (see Fig. 3(c)).

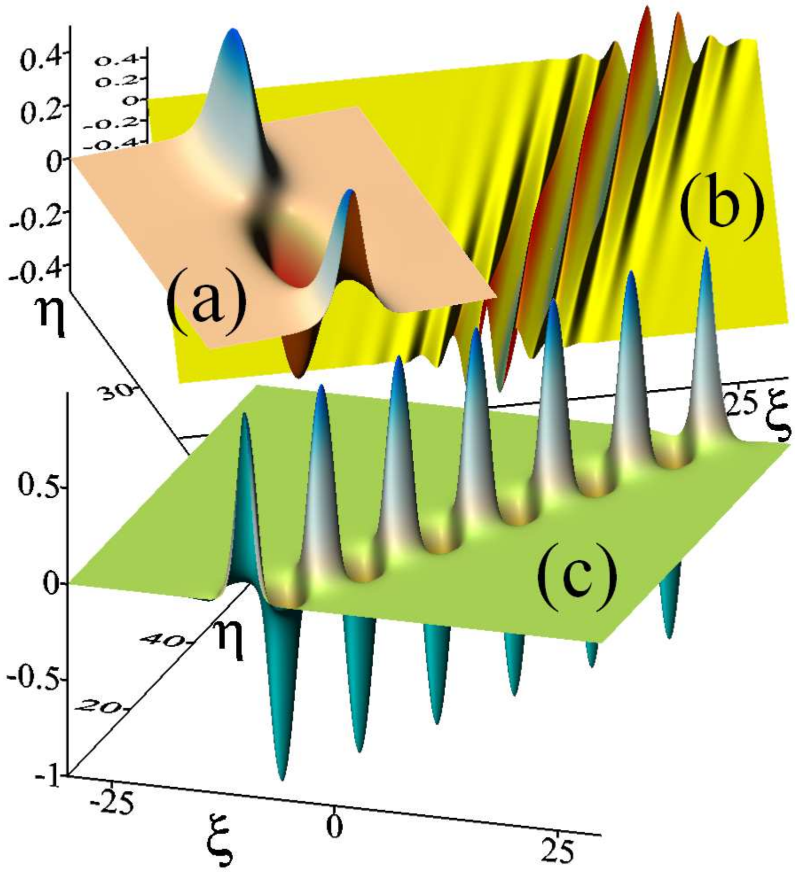


Fig. 3 Particle-wave duality of solitons. (a-c) Real parts of the soliton wave function *Re[ψ(ξ,η)]* for (a) □=*0.5* and *V=0*; (b) □=0.5 and (c) *V=2.0*; □=*1.0* and *V=0.5*.

Initially, the term "solitonic de Broglie wavelength" may seem paradoxical. All that has been said so far suggests that de Broglie's idea is connected exclusively with the Plank constant *ħ*. In his own Nobel Lecture [1], de Broglie specially emphasized that the particle-wave duality is related directly with the Galilean (in the nonrelativistic approximation) and the Lorentz symmetries. We introduce the term "solitonic de Broglie wavelength" to stress this fact that in the linear limit σ=0, Eq. (1) represents the classical SE for which the concept of "oscillating wave group as the representation of a particle in wave mechanics" was discovered by Schrödinger in 1926 [33], when the quantum mechanics was just few months old and the probabilistic interpretation of the wave function was not known yet. In one of the best papers of his "Nobel succession" [33], Schrödinger considered the harmonic oscillator model and obtained the analytical expression for the real part of the wave function Re($\psi(\xi,\eta)$) in the standard dimensionless variables [33] (that means the substitution $U(\xi)=\xi^2/2$ into the Eq. (1) with σ=0, in our variables, $\eta=\omega t$ and $\xi=x/(\hbar/m\omega)^{1/2}$, where ordinary quantum mechanical notations for a harmonic oscillator [29] have been used). Schrödinger specially emphasized that "the number and breadth of the "furrows" or "wavelets" within the particle vary with the time.

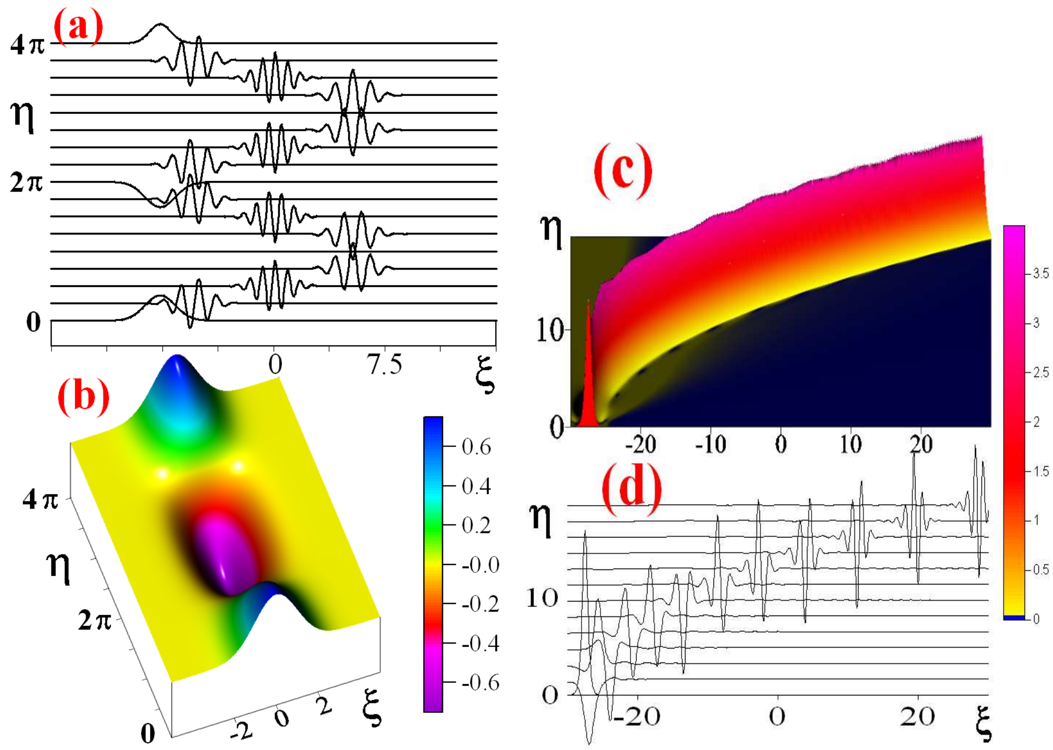


Fig. 4 "Oscillating wave group as the representation of a particle in wave mechanics" discovered by Schrödinger in 1926 [33]. (a) Real parts of the harmonic oscillator wave functions *Re[ψ(ξ,η)]*. (b) Dynamics of the ground state wave function (compare with Fig.3 (a)). (c-d) Representative examples which reveal how the Raman soliton self-scattering effect (c) brings into existence of the "solitonic de Broglie wavelength" in the (d) Real part of the wave function of colored Raman solitons.

...Our wave group always remains compact, and does not spread out into larger regions as time goes on" [33]. In Fig. 4(a-b), we illustrate this scenario (by using modern computer algorithms) to reproduce the appearance of the associated de Broglie oscillations explained for the first time in the ingenious Schrödinger's paper [33]. It is intriguing that as was shown in recent computer experiments, these "compact wave groups" interact elastically and demonstrate behavior similar to the well known interaction scenario for the NLSE solitons [34]. Schrödinger many times underlined that principle of the particle-wave duality is deeply imbedded into his equation, because (inspirited by Einstein-de Broglie ideas: $E= \hbar\omega$, $p= \hbar k$) he only tried to find the associated model which gives $E=p^2/2m$ in nonrelativistic limit [2]. Obviously, that "the particle-wave duality for solitons" is deeply imbedded into the NLSE model as well. It should be also noticed, that in modern soliton supercontinuum experiments, the appearance of the analog of the de Broglie wavelength can be easy understood by considering the effect of the soliton Raman self-frequency shift calculated in accordance with [35] and shown in Fig. 4(c-d).

During the soliton tunneling through the classically forbidden barrier $T_{kin} \le U_{max}$, the soliton kinetic energy must be transformed into the potential energy. Under the condition $\lambda_{dB} > 2\pi\xi_s$, when the soliton is stopped on the barrier, the absolute value of the soliton self-interaction energy remains be higher than the maximum value of the scattering potential $U_{max} = V^2/2$. In other words, the soliton, due to its own solitonic analog of the de Broglie wavelength, "knows its future" and, in particular, de Broglie video-soliton with $\lambda_{dB} > 2\pi\xi_s$ behaves as a classical point-like Newtonian particle (see details in Fig. 5 (a-c)).

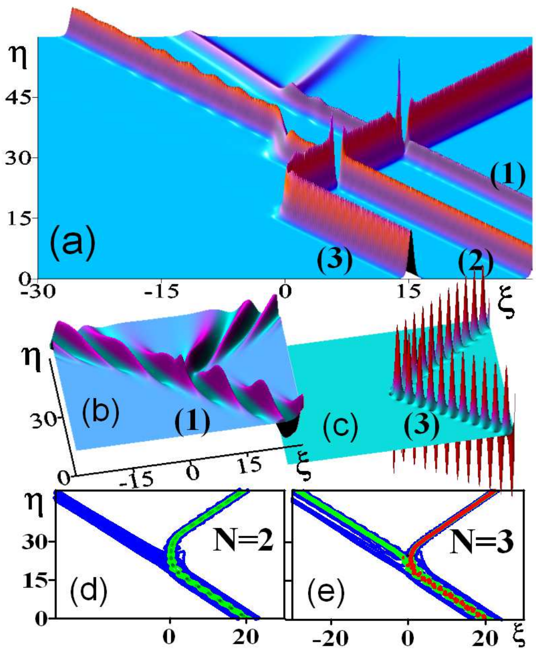


Fig. 5. Possible experimental implementations. (a) Nonlinear tunneling of the sequence of three spatial solitons through the classically forbidden parabolic barrier (corresponding to graded-index silicon antiwaveguide) $U(\xi)= U_0(1-\xi^2)$: splitting (marked as (1) $E_0>0$), sub-barrier transmission (marked as (2) $E_0 \simeq 0$) and "classical" reflection (marked as (3) $E_0<0$). (b,c) Real parts of the soliton wave function $\mathrm{Re}[\psi(\xi,\eta)]$ associated with the cases (b) $E_0>0$ and (c) $E_0<0$. (d,e) Nonlinear soliton "shooting out" effect from N-soliton bound states: (d) $N=2$ and (e) $N=3$.

There are two basic questions to be answered. What happens if arbitrary high-energy wave packet (soliton laser beam/pulse, or matter-wave soliton) approaches the classically forbidden barrier? In particular, what happens in the case of the nonlinear tunneling of the sequence of two or more isolated in space solitons with different self-interaction energies, and what happens in the opposite case of the nonlinear superposition of strongly overlapping in one space point N solitons? The understanding arises from a generalization of Eqs. (1-3) for N soliton solutions:

$$\psi(\xi,\eta) = \sum_{n=1}^{N} \Box_n \, sech[\Box_n(\xi - V_n\eta)] exp[iV_n\xi + i(\Box_n^2 - V_n^2)\eta/2) + i\varphi_{0n}], \quad (11)$$

where N is the number of solitons generated from the initial condition $\psi(\xi,\eta)=A_0 sech(\xi)$ and $\Box_n = 2(A_0 - n+1/2), n=1, 2..., N$. Each soliton in a nonlinear superposition may be characterized by its own bound-state energy level $E_n = - \Box_n{}^2/2<0$ and, consequently, under the action of the external repulsive potential U_{max}, n-soliton can be "uplifted" from its bound-state to the metastable energy level $E_n = 3/2<E_{int}>+U_{max} >0$. If only one soliton with the highest form-factor $\Box_n$ remains on the bound-state (negative) energy level E_n <0, the only this soliton can be reflected from classically forbidden potential barrier as shown in Fig. 5(d-e).

We began our article with the famous English proverb "Coming events cast their shadows before them" to stress this fact that, in the history of science, very often, outstanding scientists discovered effects which have been predecessors (shadows) of the coming future scientific paradigms. The discovery of the predecessor of the particle-wave duality dates back to 1921, when Carl Ramsauer and John Sealy Townsend investigated the scattering of low-energy electrons by atoms of a noble gases and discovered that for slow-moving electrons the probability of collision between the electrons and gas atoms obtains a minimum value at a certain kinetic energy [36-37]. From photonic point of view, the Ramsauer-Townsend effect is analogous to the Poisson (sometimes called the Arago or the Fresnel) bright spot arising at the centre of the shadow of a circular opaque obstacle. In quantum mechanics, the Ramsauer-Townsend effect corresponds to the condition when the size of a finite potential well is related to the de Broglie half wavelength by whole numbers [29]. This follows from a plane wave representation of a particle for which the reflection coefficient $R(E)$ from the square well U_0 (with full width a) as a function of the velocity V

$$R(V)= sin^2[a(V^2+2U_0)^{1/2}]/\{V^2(V^2+2U_0)/ U_0^2+sin^2[a(V^2+2U_0)^{1/2}]\} \quad (12)$$

represents sharp resonance peaks between comparatively flat minima (see Fig. 6, dark curve) varying in the limits $R(V)\to 1$ when $V\to 0$ and $R(V)\to 0$ when $V\to\infty$ [29]. The Ramsauer-Townsend effect $R(V)=0$ corresponds to the condition $a(V^2+2U_0)^{1/2}= n\pi$.

The question now arises as to whether there are significant differences in the reflection coefficient for a wave packet representation of the particle. As follows from direct computer simulations, in the linear case (σ=0 in Eq. (1)), the smaller is the width of incident wave packets, the greater is the flattening of the resonance peaks and, that is more important, resonances disappear for a very short incident packets when their energy spectra are so wide that overlap two or more resonance levels inside the potential well.

What happens with a soliton when it is scattered by a square well? At small soliton form-factors, soliton practically has no action upon the scattering potential. At the certain value of kinetic energy of the incoming soliton, the reflection coefficient rises its minima and the solitonic Ramsauer-Townsend effect can be observed (see Figs.6-7). The Ramsauer-Townsend transparency for solitons takes place when the potential width $a=n\,\lambda_{dB}'/2$ equals an integral number of "solitonic de Broglie half-wavelengths" inside the well, where $\lambda_{dB}'=2\pi\,(V^2+2\tilde{U}_0)^{1/2}$. Essentially no energy is lost during the solitonic Ramsauer-Townsend effect as shown in Fig. 6.

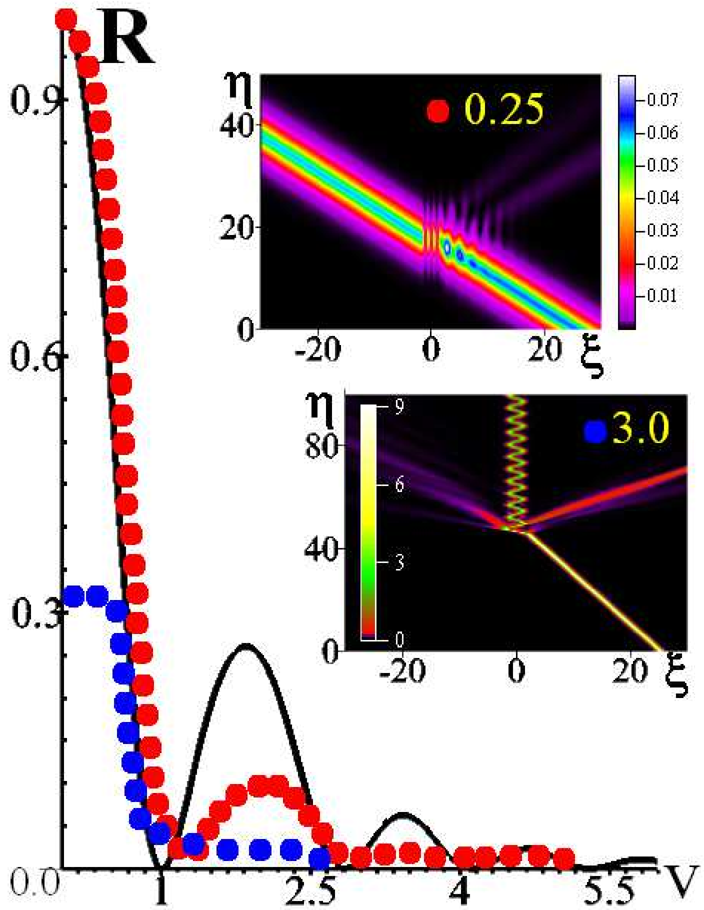
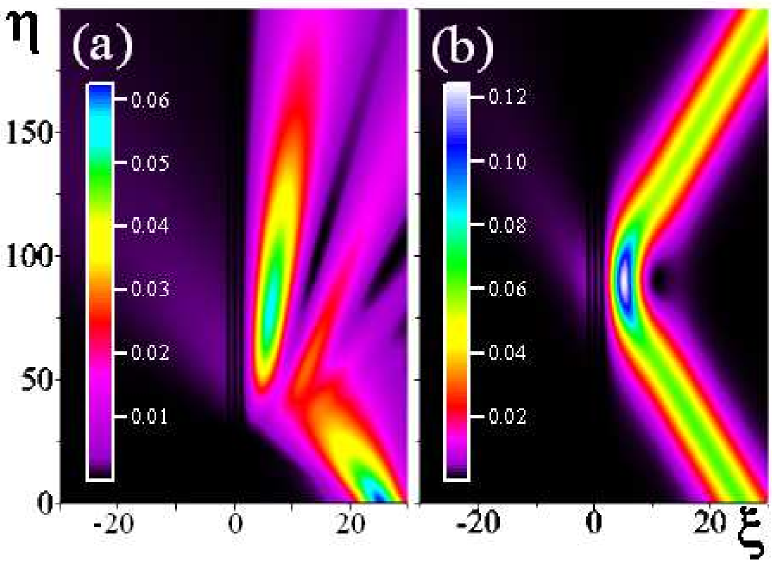


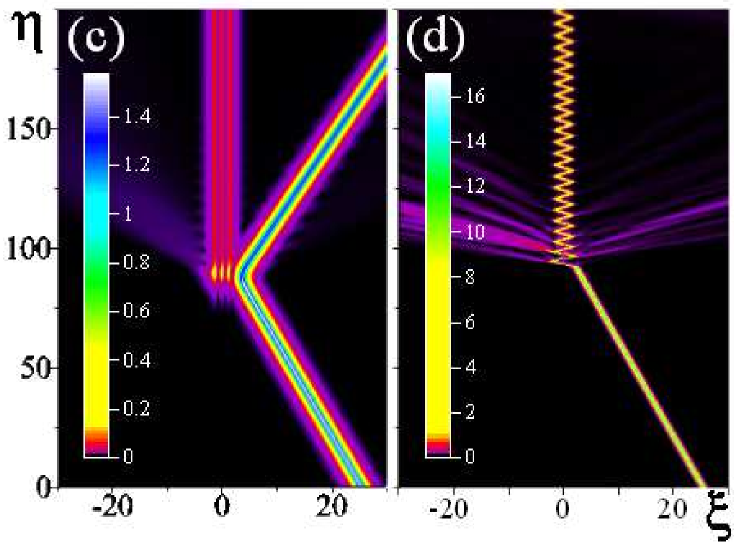
Fig. 6. Nonlinear (solitonic) Ramsauer-Townsend effect. Reflection coefficients *R(E)* from the square-well potential with the depth U_0=4.0 and width $a=\pi/2$ after choosing different scattering regimes: dark

curve corresponds to the linear scattering; red to the soliton scattering with form-factor □=*0.25* and $V\tilde{=}1.4$; blue to the soliton scattering with form-factor □=*3.0* and $V\tilde{=}0.5$.

All the most interesting features of the low energy soliton scattering by an attractive potential are summarized in Fig. 7(a-h). In Fig. 7(a,b), we compare linear (σ=0 in Eq. (1)) and nonlinear (σ=1 in Eq. (1)) scenarios of scattering of low energy wave packets with kinetic energies $E << U_0$. In both cases, the interference structures inside the well decay with time, that is physically obvious result. This reflection from an attractive potential is a result of the wave nature of solitons. At the larger soliton form-factors □, the soliton behavior is very different from the "classical quantum mechanical" one: the effective trapping potential $U_{eff}\tilde{=}U(\tilde{\xi}|\psi(\xi,\eta)|^2$ is affected considerably by the solitonic own self-trapping potential and, because of the particular trapping of energy inside the well (see Fig. 7(c-d)), the reflected coefficient transforms into the step like plateau represented in Fig. 6 by blue circlets. The "hidden" role of the soñiton self-interaction energy in the particle-wave duality of scattered solitons comes into particular prominence during comparison all the cases of higher soliton form-factors shown in Fig. 7(e-f). In particular, Fig. 7(e) presents the case when the Newtonian-like "particle" oscillates inside the trapping potential. There exists the critical strength of the soliton self-interaction energy above which all solitons are transmitted and never reflected (see Fig. 7(f)). In other words, the fact that the soliton with high self-interaction ("binding") energy resembles a classical particle has become apparent. The physical explanation of this lies in the fact that the greater is the soliton amplitude □ the lower is associated with a soliton its own bound-state energy level $E_n\tilde{=}$ □$_n^2/2<0$ given by Eq. (5), and this level does not overlap with no other resonance level of the trapping potential. This effect takes place when

$\tilde{}$ □$_n^2/2 \tilde{<} U_{max}$ or □$_0>(2U_{max})^{1/2}$.


η
(a)
(b)
0.06
0.05
0.04
0.03
0.02
0.01
0.12
0.10
0.08
0.06
0.04
0.02
150
100
50
0
-20
0
20
ξ


η
(c)
(d)
1.4
1.2
1
0.8
0.6
0.4
0.2
0
16
14
12
10
8
6
4
2
0
150
100
50
0
-20
0
20
ξ

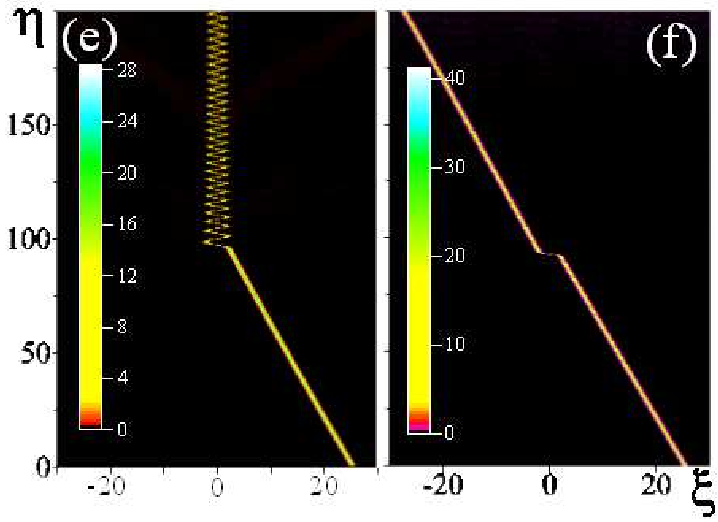


Fig. 7. "Hidden" features of the solitonic Ramsauer-Townsend effect. (a,b) The comparison of linear (quantum mechanical like) wave packet reflection from the attractive potential (a) and nonlinear soliton reflection (b) with the parameters □*=0.25* and *V=0.25*. (c) Nonlinear trapping of the soliton energy inside the well in the parameters region □*=0.25* and *V=0.25*. (d,e) Soliton oscillations inside the attractive well in the parameters regions *(*d)□*=4.0* and (e) □*=5.0*. (f) The existence of the critical soliton form-factor □*=6.0* (critical strength of the soliton self-interaction energy given by Eq.(9)) above which all solitons are transmitted and never reflected in complete agreement with behaviors of classical Newtonian particles.

Understanding of the leading role of the soliton self-interaction energy in the particle-wave duality of solitons can find many applications, for example, in the developing of novel all-optical soliton logic and switching devices, in soliton lasers design, for soliton supercontinuum generation and formation of matter wave solitons in BEC. Let us consider possible experimental implementations. In nonlinear optical applications, the silicon graded-index planar structure [38] can be proposed which is only 1 *μkm* thick and confines input beam in the y direction. No such confinement occurs in the x direction along which the refractive index varies as $n(x,I)=n_0+ n_1x^2+ n_2I$. If we use typical parameters for silicon $n_2=6\cdot10^{-14}$cm²/W near $\lambda_0=1.55$ *μkm*, we obtain the required peak intensity about 30 MW/cm² for the spatial soliton with characteristic dimensions 100μkm × 1μkm which translates into power levels 30 W. To observe the effects presented in Fig. 5, the refractive index must be graded with $n_1=1.8\cdot10^{-2}$cm^{-2}.

The results reported in this work are of general physics interest and offer many opportunities for further scientific studies: although many different types of solitons have been already discovered in different branches of science (today, approximately one hundred models have been found to have soliton-like solutions), the understanding of the leading role of the soliton self-interaction energy in their dynamics still remains to be "hidden" and continue to open up unexpected directions of study. The interpenetration of the main ideas and methods being used in different fields of science and technology becomes at present one of the decisive factors for the progress of science as a whole. In particular, since related dynamics governed by the NLSE model are also observed in many other physical systems from plasmas and BECs to monster (rogue) waves in oceans, our results are expected to stimulate new research directions in many other fields [17,21,38,39].

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[1] de Broglie, L. The wave nature of the electron. Nobel Lecture, December 12, 1929, 244-256; Biography, 257-259 (World Scientific, 1998).

[2] Schrödinger, E. The fundamental idea of wave mechanics. Nobel Lecture, December 12, 1933, 305-316; Biography, 317-319 (World Scientific, 1998).

[3] Rebbi, C., & Soliani, G. Solitons and Particles (Word Scientific, Singapore, 1984).

[4] Manton, N. S. Solitons as elementary particles: a paradigm scrutinized. Nonlinearity 21, T221̃T232 (2008).

[5] Drazin, P.G. & Jonson, R.S. Solitons: an Introduction (Cambridge University Press, 1989).

[6] Taylor, J. R. Optical solitons: theory and experiments (Cambridge University Press, 1992)

[7] Hasegawa A. (Ed.), Massive WDM and TDM Soliton Transmission Systems (Kluwer Academic Publishers, Dordrecht/Norwell, MA, 2000).

[18] Dudley, J. M. & Taylor, J. R. Ten years of nonlinear optics in photonic crystal fibre. Nature Photonics, 3, 85-95 (2009).

[9] Dudley, J. M. & Taylor, J. R. Optical Fiber Supercontinuum Generation (Cambridge University Press, Cambridge, 2010).

[10] Stegeman, G. I. & Segev, M. Optical spatial solitons and their interactions: Universality and diversity. Science 286, 1518-1523 (1999).

[11] Christodoulides, D. N., Lederer F., & Silberberg, Y. Discretizing light behavior in linear and nonlinear waveguide lattices. Nature 424, 817-823 (2003).

[12] Gauthier, D. J. Solitons go slow. Nature Photonics 1, 92-93 (2007).

[13] Dudley, J. M. Ultrafast optics: Nonlinear attraction. Nature Photonics 4, 471-476 (2010).

[14] Colman, P., Husko, C., Combrie, S. et al. Temporal solitons and pulse compression in photonic crystal waveguides. Nature Photonics 4, 862-868 (2010).

[15] Won, R. View from... Nonlinear photonics 2010: Revisiting solitons. Nature Photonics 4, 591-592 (2010).

[16] To avoid confusion, we stress that exclusively nonintegrable models are being considered and investigated in this article. The exactly integrable nonautonomous models with external potentials are summarized in: Serkin, V.N., Hasegawa, A. & Belyaeva, T. L. Nonautonomous solitons in external potentials. Phys. Rev. Lett. 98, 074102 (2007); He, X.-G., Zhao, D., Li, L. & Luo, H.-G. Engineering integrable nonautonomous nonlinear Schrödinger equations. Phys. Rev. E 79, 056610 (2009); Serkin, V. N., Hasegawa, A. & Belyaeva, T. L. Solitary waves in nonautonomous nonlinear and dispersive systems: nonautonomous solitons. Journal of Modern Optics, 57, 1456-1472 (2010).

[17] Salasnich, L. Pulsed quantum tunneling with matter waves. Laser Physics, 12, 198--202 (2002).

[18] Stoychev, K. T., Primatarowa, M. T. & Kamburova, R. S. Resonant scattering of nonlinear Schrödinger solitons from potential wells, Phys. Rev. E 70, 066622 (2004).

[19] Peccianti, M., Daydyusha, A., Kaczmarek, M. & Assanto, G. Tunable refraction and reflection of self-confined light beams. Nature Physics 2, 737-742 (2006).

[20] Linzon, Y., Morandotti, R., Volatier, M., Aimez, V., Ares, R. & Bar-Ad, S. Nonlinear scattering and trapping in bylocal photonic potentials. Phys. Rev.Lett. 99, 133901 (2007).

[21] Demidov, V. E., Hansen, U. H. & Demokritov, S. O. Experimental observation of enhanced interaction of magnetic solitons with potential barriers and wells. Phys. Rev. B 78, 054410 (2008).

[22] Yang, R. & Wu, X. Spatial soliton tunneling, compression and splitting. Opt. Express 16, 17759 (2008).

[23] Barak, A., Peleg, O., Stucchio, C., Soffer, A. & Segev, M. Observation of soliton tunneling phenomena and soliton ejection. Phys. Rev. Lett. 100, 153901 (2008).

[24] Wan, W., Muenzel, S. & Fleischer, J. W. Wave tunneling and hysteresis in nonlinear junctions. Phys. Rev. Lett. 104, 073903 (2010).

[25] Lemasson, A., Shrivastava, A., Rejmund, M., Keeley, N., Zelevinsky, V., Bhattacharyya, S., Chatterjee, A., de France, G., Jacquot, B., Nanal, V., Pillay, R. G., Raabe, R. & Schmitt, C. Modern Rutherford experiment: tunneling of the most neutron-rich nucleus. Phys. Rev. Lett., 103, 232701 (2009).

[26] Hasse, R.W. Schrödinger solitons and kinks behave like Newtonian particles. Phys. Rev. A 25, 583 (1982).

[27] de Moura, M.A. Newtonian behavior of Schrödinger solitons and kinks. Phys. Rev. A 37, 4998 (1988).

[28] In BEC physics, Eq.(2) is known also as the Gross-Pitaevskii equation after Tsuzuki, T. J. Low Temp. Phys. 4, 441 (1971).

[29] Merzbacher, E. Quantum Mechanics, Third Edition (John Wiley & Sons, Inc., New York, 1998).

[30] Gamow, G. Quantum Theory of the Atomic Nucleus. Z. Phys. 51, 204-212 (1928).

[31] Pöschl G. & Teller, E. Bemerkungen zur Quantenmechanik des anharmonischen Osziltators. Z. Phys. 83, 143-151 (1933); Kay I. & Moses, H. E. Reflectionless transmission through dielectrics and scattering potentials. J. Appl. Phys. 27, 1503--1508 (1956).

[32] McKagan, S. B., Perkins, K. K.,& Wieman, C. E. A Deeper Look at Student Learning of Quantum Mechanics: the Case of Tunneling. Phys. Rev. ST: Phys. Educ. Res., 4, 020103 (2008).

[33] Schrödinger, E. Naturwissenschaften 14, 664 (1926). English Translation in: The continuous transition from Micro- to Macro-Mechanics, Collected papers on wave mechanics, 41-44 (Chelsea Publishing Company, New York, 1978).

[34] Hernandez, T.C., Villagran, V.E., Serkin, V. N. et al. Dynamics of solitons in the model of nonlinear Schrodinger equation with an external harmonic potential: I. Bright solitons. Quantum Electron. 2005, 35, 778--786 (2005); II. Dark solitons. ibid. 35, 929-938 (2005).

[35] Dianov, E. M., Mamyshev, P. V., Prokhorov, A. M. & Serkin, V. N. Nonlinear Effects in Fibers (Harwood Academic: New York, 1989); Belyaeva, T. L., Serkin, V. N., Hernandez-Tenorio, C. &. Garcia-Santibanez, F. Enigmas of optical and matter-wave soliton nonlinear tunneling. Journal of Modern Optics 57, 1087--1099 (2010).

[36] Ramsauer, C. Über den Wirkungsquerschnitt der Gasmoleküle gegenüber langsamen Elektronen. Annalen der Physik, 4, 513---540 (1921).

[37] Bailey, V. A. & Townsend, J. S. The motion of electrons in gases. Philosophical Magazine, 6, 873-891 (1921).

[38] Leuthold, J., Foos, C. & Freude, W. Nonlinear silicon photonics. Nature Photonics 4, 535-544 (2010).

[39] Solli, D. R., Ropers, C., Koonath, P.& Jalali, B. Optical rogue waves. Nature, 450, 1054 (2007).